

## References and Supplemental Reading

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10. Wisdom, J. Urey Prize Lecture: "Chaotic Dynamics in the Solar System" (1987) Icarus 72, pp. 241-275.
11. Taff, L.G., "Celestial Mechanics: "A computational guide for the practitioner" (1985) John Wiley & Sons.

The references above constitute required reading for any who would become a practitioner of celestial mechanics. Certainly Morse and Feshbach is one of the most venerable texts on theoretical physics and contains more information than most theoreticians would use in a lifetime. However, the book should be in the arsenal that any theoretician brings to the problems of analysis in physics. I still feel that Goldstein's text on classical mechanics is the best and most complete of the current era. However, some may find the text by Symon somewhat less condensed. The text by Brouwer and Clemence is the most advanced of the current texts in the field of celestial mechanics and is liable to remain so for some time to come. It is rather formidable, but contains information on such a wide range of problems and techniques that it should be at least perused by any student of the field. The text by Danby was the logical successor to the time honored work of Moulton. Danby introduced vector notation to the subject and made the reading much simpler. A.E. Roy expanded on this approach and covered a much wider range of topics. The celestial mechanics text by Fitzgerald listed below provides a development more common to modern day celestial mechanics and contains an emphasis on the orbital mechanics of satellites. This point of view is also used by Escobal where the first book on the "Methods of Orbit Determination" lays the groundwork for a contemporary discussion of 'rocket navigation' in the second book on "Astrodynamics". A much broader view of the term astrodynamics is taken by Herrick in his two volume treatise on the subject. The five volume 'epic' by Hagihara tries to summarize all that has happened in celestial mechanics in the last century and comes close to doing so. The text by Taff is one of the most recent of the celestial mechanics texts mentioned here, but still largely follows the traditional development started by Moulton. The exception is his discussion of perturbation theory which I found philosophically satisfying. The Urey Prize lecture by Wisdom should be read in its entirety by anyone who is interested in the application of the mathematics of chaos to objects in the solar system.

Below I have given some additional references as 'supplemental reading' which I have found helpful from time to time in dealing with the material covered in this book. Most any book on modern algebra will contain definitions of what constitutes a set or group, any book on modern algebra will contain definitions of what constitutes a set or group, but I found Andree very clear and concise. One of the best all round books on mathematical analysis with a view to numerical applications is that by Arfken. It is remarkably complete and wide ranging. The two articles from Chaotic Phenomena in Astrophysics show some further application of the subjects discussed by Wisdom. However, the entire book is

interesting as it demonstrates how this developing field of mathematics has found applications in a number of areas of astrophysics.

Sokolnikoff and Redheffer is just one of those omnibus references that provide a myriad of definitions and development for mathematical analysis necessary for any student of the physical sciences. On the other hand, the lectures by Ogorodnikov provide one of the most lucid accounts of Liouville's Theorem and the implications for a dynamical system in phase space. The text on Gravitation by Misner, Thorne, and Wheeler has probably the most contemporary and complete treatment of tensors as they apply to the physical world. Although the main subject is somewhat tangent to celestial mechanics, it is a book that every educated physicist or astrophysicist must read. Since it is rather long, one should begin early. One should not leave the references of celestial mechanics without a mention of the rare monograph by Paul Herget. While the presentation of the material is somewhat encumbered by numerical calculations for which Paul Herget was justly renowned, the clarity of his understanding of the problems of classical orbit calculation makes reading this work most worthwhile.

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